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A wait-and-see strategy as a survival strategy in the prisoner's dilemma between relatives

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Abstract. The behavior to decide the action according to the current situation is seen well in humans. We consider the prisoner's dilemma in which the variable probability of cooperation is allowed. Here, we define a wait-and-see strategy as the strategy that an individual cooperates at the same probability as the proportion of cooperation in the population. In addition, we consider a game between relatives in which an individual is more likely to meet an opponent using the same strategy. To examine the reasonableness of a wait-and-see strategy from the viewpoint of survival, we analyze the three strategies in the prisoner's dilemma between relatives by means of a replicator dynamics. We prove that our wait-and-see strategy survives in almost all conditions for the prisoner's dilemma between relatives. Therefore, we conclude a wait-and-see behavior is reasonable.

Keywords. wait-and-see strategy, prisoner's dilemma, game between relatives, survival, replicator dynamics

1. Introduction

The behavior to decide the action according to the situation is seen well in humans. In this paper, we consider a strategy that depends on the current situation in the prisoner's dilemma. We define a wait-and-see strategy as the strategy that an individual cooperates at the same probability as the proportion of cooperation in the population. Many authors investigated the prisoner's dilemma in which variable probabilities of cooperation is allowed (Nowak [9]; Verhoeff [16]; Frean [2]; Doebeli and Knowlton [1]; Killingback et al. [8]; Wahl and Nowak [18], [19]; Killingback and Doebeli [7]). They proposed various mechanisms that cooperation can emerge in the prisoner's dilemma. However, in these models, the probability of cooperation depends either on previous payoffs or on an opponent's previous action. In contrast, a wait-and-see strategy depends on the current situation, but does not bring the emergence of cooperation. Furthermore, a wait-and-see strategy is not rational. Nevertheless we can often see humans doing a wait-and-see behavior.

Evolutionary game theory, replicator dynamics, implicitly assumes that all individuals are randomly matched (Taylor and Jonker [14]; Weibull [20]; Hofbauer and Sigmund [5], [6]). On the other hand, there exist some researches relaxing this assumption (Grafen [3]; Hines and Maynard Smith [4]; Thomas [15]; Taylor [13]; Vincent and Cressman [17]; Tao and Lessard [10]; Taylor and Nowak [11]). Grafen [3], Hines and Maynard Smith [4] studied games between relatives. Grafen proposed a concept of "personal fitness" and introduced this concept into the hawk-dove game. The "personal fitness" approach modifies the fitness of an individual by allowing for the fact that an individual is more likely to meet an opponent using the same strategy. If we introduce the concept of "personal fitness" into the prisoner's dilemma, cooperation can emerge. Indeed, the cooperator will not intend to approach the defector in the prisoner's dilemma. Therefore, it seems realistic to relax the assumption that all individuals are randomly matched. However, neither cooperation nor defection is guaranteed, regardless of the conditions for the prisoner's dilemma between relatives. It is natural to ask whether there exists a strategy that survives in all conditions.

For these reasons, to examine the reasonableness of a wait-and-see strategy from the viewpoint of survival, we analyze the three strategies (cooperation, defection and a wait-and-see strategy) in the prisoner's dilemma between relatives by means of a replicator dynamics. As a result, we show that a wait-and-see strategy survives in almost all conditions for the prisoner's dilemma between relatives. In this sense, we can say that the behavior to cooperate at the same probability as the proportion of cooperation in the population is reasonable.

The organization of this paper is as follows: In section 2, we state the "personal fitness" approach, proposed by Grafen [3], and give a preliminary result. In section 3, we define a wait-and-see strategy and prove that a wait-and-see strategy survives in almost all conditions for the prisoner's dilemma between relatives. Finally, in section 4 we conclude with some discussions.

2. The prisoner's dilemma between relatives

In this section, we state the prisoner's dilemma between relatives, and in the framework of evolutionary game theory, see that cooperation can emerge in this game.

In the prisoner's dilemma, the available strategies are cooperation and defection. The payoff matrix is given by

$$A = \begin{bmatrix} R & S \\ T & P \end{bmatrix}.$$

If both individuals cooperate, they have payoff R. If one individual cooperates while the other individual defects, the cooperator receives S, and the defector receives T. If both individuals defect, they have P. The prisoner's dilemma is defined by the conditions $T > R > P > S$ and $2R > T+S$.

Let x_C and x_D be the proportion of the population cooperating and defecting at time t, respectively. Then, we have $x_C, x_D \geq 0$, and $x_C + x_D = 1$. Following Taylor and Jonker [14], Weibull [20], Hofbauer and Sigmund [5], [6], we can write the evolutionary dynamics by the replicator equation:

$$\begin{aligned} \dot{x}_C &= (f_C - f_D)x_D x_C, \\ \dot{x}_D &= (f_D - f_C)x_C x_D, \end{aligned}$$

where f_C and f_D denote the fitness of cooperators and defectors, respectively. If we assume that all individuals are randomly matched, the fitness f_C and f_D are given by

$$\begin{aligned} f_C &= Rx_C + Sx_D, \\ f_D &= Tx_C + Px_D. \end{aligned}$$

Therefore, we have $x_C \to 0$ as $t \to \infty$.

Now, we suppose that all individuals are not randomly matched. Grafen [3] proposed a concept of "personal fitness" to account for the fact that an individual is more likely to meet an opponent with the same strategy. This game is called a game between relatives. In Grafen's model, the fitness of cooperators f_C and defectors f_D are given by

$$\begin{aligned} f_C &= rR + (1-r)(Rx_C + Sx_D), \\ f_D &= rP + (1-r)(Tx_C + Px_D), \end{aligned}$$

where r is the probability that an individual meets an opponent with the same strategy because they are related, and $1-r$ is the probability that all individuals are randomly matched. Since

$$\begin{aligned} & f_C - f_D \\ &= r(R-P) + (1-r)((R-T)x_C + (S-P)x_D) \\ &= r(R-S) - (P-S) + (1-r)(R+P-T-S)x_C, \end{aligned}$$

we have

$$\text{(1)} \quad \begin{aligned} \dot{x}_C = \{&r(R-S) - (P-S) \\ &+ (1-r)(R+P-T-S)x_C\}(1-x_C)x_C. \end{aligned}$$

Let

$$q = \frac{(P-S) - r(R-S)}{(1-r)(R+P-T-S)}.$$

Then, we have the following result from (1).

If $x_C(0) = 0$ or 1, we have $x_C(t) = 0$ or 1, respectively. Therefore, we consider the case where $0 < x_C(0) < 1$.

(1) The case where $R+P-T-S > 0$.
(i) If $r \leq (T-R)/(T-P)$, we have $\lim_{t\to\infty} x_C(t) = 0$.
(ii) If $(P-S)/(R-S) \leq r$, we have $\lim_{t\to\infty} x_C(t) = 1$.
(iii) If $(T-R)/(T-P) < r < (P-S)/(R-S)$, we have the following result.
When $x_C(0) < q$, we have $\lim_{t\to\infty} x_C(t) = 0$.
When $x_C(0) > q$, we have $\lim_{t\to\infty} x_C(t) = 1$.
When $x_C(0) = q$, we have $x_C(t) = x_C(0) = q$.

(2) The case where $R+P-T-S < 0$.
(i) If $r \leq (P-S)/(R-S)$, we have $\lim_{t\to\infty} x_C(t) = 0$.
(ii) If $(T-R)/(T-P) \leq r$, we have $\lim_{t\to\infty} x_C(t) = 1$.
(iii) If $(P-S)/(R-S) < r < (T-R)/(T-P)$, we have $\lim_{t\to\infty} x_C(t) = q$.

(3) The case where $R+P-T-S = 0$.
(i) If $r < (P-S)/(R-S)$, we have $\lim_{t\to\infty} x_C(t) = 0$.
(ii) If $(P-S)/(R-S) < r$, we have $\lim_{t\to\infty} x_C(t) = 1$.
(iii) If $r = (P-S)/(R-S)$, we have $x_C(t) = x_C(0)$.

3. Survival strategy in the prisoner's dilemma between relatives

In this section, we define a wait-and-see strategy, and analyze the three strategies (cooperation, defection and a wait-and-see strategy) in the prisoner's dilemma between relatives by means of a replicator dynamics.

Here, we introduce a new strategy.

Definition 1. We define a wait-and-see strategy as the strategy that an individual cooperates at the same probability as the proportion of cooperation in the population.

We extend the prisoner's dilemma. In this game, we introduce a wait-and-see strategy as the third strategy. Therefore, there are three allowed strategies: (1) always cooperating, (2) always defecting, and (3) using a wait-and-see strategy. Let x, y and z be the proportion of the population always cooperating, always defecting and using a wait-and-see strategy at time t, respectively. Then, we have $x, y, z \geq 0$ and $x+y+z = 1$. From now on, we suppose that $0 < z(0) < 1$. Let x_C and x_D be the proportion of the cooperation and defection in the population at time t, respectively. Then, we have $x_C, x_D \geq 0$, $x_C + x_D = 1$, and $x_C = x + x_C z$. Hence, we have

$$x_C = \frac{x}{1-z} = \frac{x}{x+y}.$$

Therefore, we have

$$\begin{aligned}\dot{x}_C &= \frac{\dot{x}y - x\dot{y}}{(x+y)^2} \\ &= \left(\frac{\dot{x}}{x} - \frac{\dot{y}}{y}\right)\frac{xy}{(x+y)^2} \\ &= (f_{AC} - f_{AD})(1-x_C)x_C,\end{aligned}$$

where f_{AC} and f_{AD} denote the fitness of the individual always cooperating and always defecting, respectively.

Now, we assume that all individuals are not randomly matched. Let X and Y be stochastic process that take values in $\mathcal{S}_3 = \{AC, AD, WS\}$, where the strategies AC, AD and WS denote always cooperating, always defecting and a wait-and-see strategy, respectively. Let $P(X = AC) = P(Y = AC) = x$ and so on. In this case, we assume that the distribution of stochastic process (X, Y) is decided depending on the distribution of X and six parameters. Namely, we suppose that

$$\begin{aligned}P(Y = AD \mid X = AC) &= (1-r_1)y, \\ P(Y = WS \mid X = AC) &= (1-r_2)z, \\ P(Y = AC \mid X = AD) &= (1-r_3)x, \\ P(Y = WS \mid X = AD) &= (1-r_4)z, \\ P(Y = AC \mid X = WS) &= (1-r_5)x, \\ P(Y = AD \mid X = WS) &= (1-r_6)y,\end{aligned}$$

where r_i $(i = 1, \ldots, 6)$ does not depend on t. Hence, we have

$$\begin{aligned}P(Y = AC \mid X = AC) &= x + r_1 y + r_2 z, \\ P(Y = AD \mid X = AD) &= y + r_3 x + r_4 z, \\ P(Y = WS \mid X = WS) &= z + r_5 x + r_6 y.\end{aligned}$$

Therefore, Grafen's model corresponds to the special case where $r_1 = \cdots = r_6 (= r)$.

In Grafen's model, the fitness of an individual always cooperating f_{AC} and always defecting f_{AD} are given by

$$\begin{aligned}f_{AC} &= rR + (1-r)(Rx_C + Sx_D), \\ f_{AD} &= rP + (1-r)(Tx_C + Px_D).\end{aligned}$$

Then, we have

$$\begin{aligned}(2) \qquad \dot{x}_C = \{&r(R-S) - (P-S) \\ &+ (1-r)(R+P-T-S)x_C\}(1-x_C)x_C.\end{aligned}$$

Therefore, we obtain the same equation as (1) which is obtained in the case of 2×2 game (the case where the available strategies are only cooperation and defection). Note that (2) implies that with the probability r an individual cooperating at time t meets an opponent cooperating at time t, regardless of always cooperating or using a wait-and-see strategy, and with the probability $1-r$ all individuals are randomly matched.

We denote by f_{WS} the fitness of an individual using a wait-and-see strategy. Then, we have

$$\begin{aligned}f_{WS} &= r\{(Rx_C + Sx_D)x_C + (Tx_C + Px_D)x_D\} \\ &\quad + (1-r)\{(Rx_C + Tx_D)x_C + (Sx_C + Px_D)x_D\} \\ &= Rx_C^2 + Sx_Cx_D + Tx_Cx_D + Px_D^2.\end{aligned}$$

Hence, we have

$$\begin{aligned}(3) \qquad \dot{z} &= \{(f_{WS} - f_{AC})x + (f_{WS} - f_{AD})y\}z \\ &= -r(R+P-T-S)(1-x_C)x_C(1-z)z.\end{aligned}$$

Therefore, we have the following theorem.

Theorem 1. *A wait-and-see strategy survives in the prisoner's dilemma between relatives, except for the case where all of the following three conditions are fulfilled;*

(i) $R + P - T - S > 0$,

(ii) $\dfrac{T-R}{T-P} < r < \dfrac{P-S}{R-S}$,

(iii) $x_C(0) = q = \dfrac{(P-S) - r(R-S)}{(1-r)(R+P-T-S)}$.

To prove Theorem 1, we only have to show the following proposition.

Proposition 1. *If we introduce a wait-and-see strategy as the third strategy into the prisoner's dilemma between relatives, we have the following result.*

(0) *If* $x_C(0) = 0$ *or* 1, *we have* $z(t) = z(0)$.

From now on, we consider the case where $0 < x_C(0) < 1$.

(1) *The case where* $R + P - T - S > 0$.
(i) *If* $r \le (T-R)/(T-P)$, *we have* $\lim_{t\to\infty} x_C(t) = 0$ *and* $\lim_{t\to\infty} z(t) \ne 0$ (*see Fig.* 1).
(ii) *If* $(P-S)/(R-S) \le r$, *we have* $\lim_{t\to\infty} x_C(t) = 1$ *and* $\lim_{t\to\infty} z(t) \ne 0$ (*see Fig.* 2).
(iii) *If* $(T-R)/(T-P) < r < (P-S)/(R-S)$, *we have the following result* (*see Fig.* 3). *When* $x_C(0) < q$, *we have* $\lim_{t\to\infty} x_C(t) = 0$ *and* $\lim_{t\to\infty} z(t) \ne 0$. *When* $x_C(0) > q$, *we have* $\lim_{t\to\infty} x_C(t) = 1$ *and* $\lim_{t\to\infty} z(t) \ne 0$. *When* $x_C(0) = q$, *we have* $x_C(t) = q$ *and* $\lim_{t\to\infty} z(t) = 0$.

(2) *The case where* $R + P - T - S < 0$.
(i) *If* $r \le (P-S)/(R-S)$, *we have* $\lim_{t\to\infty} x_C(t) = 0$ *and* $\lim_{t\to\infty} z(t) \ne 0$ (*see Fig.* 4).
(ii) *If* $(T-R)/(T-P) \le r$, *we have* $\lim_{t\to\infty} x_C(t) = 1$ *and* $\lim_{t\to\infty} z(t) \ne 0$ (*see Fig.* 5).
(iii) *If* $(P-S)/(R-S) < r < (T-R)/(T-P)$, *we have* $\lim_{t\to\infty} x_C(t) = q$ *and* $\lim_{t\to\infty} z(t) = 1$ (*see Fig.* 6).

(3) *The case where* $R + P - T - S = 0$. *In this case,* $z(t) = z(0)$.
(i) *If* $r < (P-S)/(R-S)$, *we have* $\lim_{t\to\infty} x_C(t) = 0$.
(ii) *If* $(P-S)/(R-S) < r$, *we have* $\lim_{t\to\infty} x_C(t) = 1$.
(iii) *If* $r = (P-S)/(R-S)$, *we have* $x(t) = x(0)$ *and* $y(t) = y(0)$.

Proof. See section 5. □

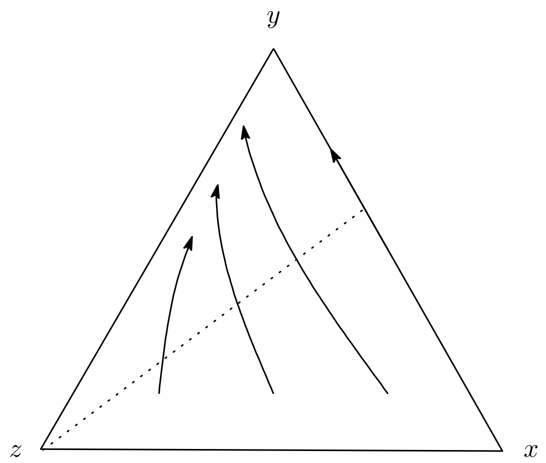

Figure 1: The case where $R+P-T-S>0$ and $r \leq (T-R)/(T-S)$. In this case, for any $0<v_0, v_1, v_2<1$ $(v_1 \neq v_2)$, two initial points $(x_C(0), z(0)) = (v_0, v_1)$ and (v_0, v_2) converge to different points, respectively (the dotted line corresponds to $x_C = v_0$).

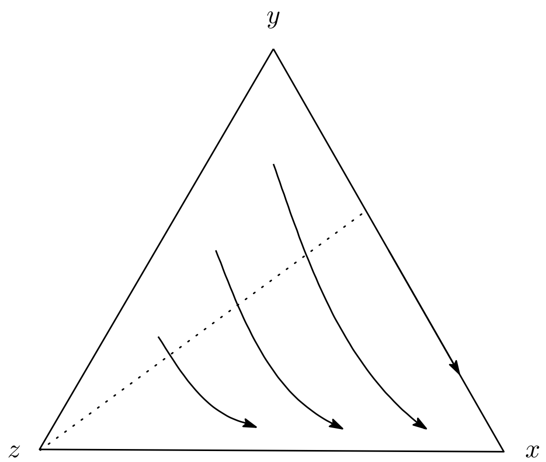

Figure 2: The case where $R+P-T-S>0$ and $(P-S)/(R-S) \leq r$. In this case, for any $0<v_0, v_1, v_2<1$ $(v_1 \neq v_2)$, two initial points $(x_C(0), z(0)) = (v_0, v_1)$ and (v_0, v_2) converge to different points, respectively (the dotted line corresponds to $x_C = v_0$).

4. Conclusion

In this paper, to examine the reasonableness of a wait-and-see strategy from the viewpoint of survival, we analyze the three strategies (cooperation, defection and a wait-and-see strategy) in the prisoner's dilemma between relatives by means of a replicator dynamics. The behavior to decide the action according to the current situation is seen well in humans. We define a wait-and-see strategy as the strategy

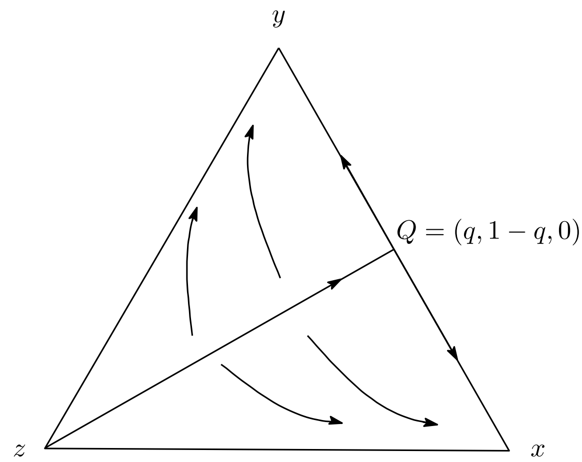

Figure 3: The case where $R+P-T-S>0$ and $(T-R)/(T-S) < r < (P-S)/(R-S)$. In this case, for any $0<v_0, v_1, v_2<1$ $(v_0 \neq q,\ v_1 \neq v_2)$, two initial points $(x_C(0), z(0)) = (v_0, v_1)$ and (v_0, v_2) converge to different points, respectively. Any initial point with $x_C(0) = q$ converges to $(x, y, z) = (q, 1-q, 0) (= Q)$ on $x_C = q$.

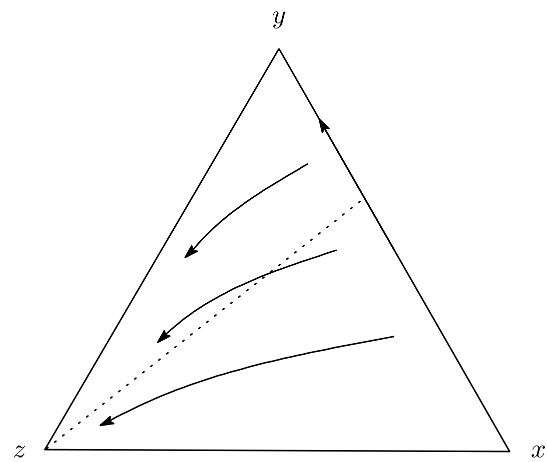

Figure 4: The case where $R+P-T-S<0$ and $r \leq (P-S)/(R-S)$. In this case, for any $0<v_0, v_1, v_2<1$ $(v_1 \neq v_2)$, two initial points $(x_C(0), z(0)) = (v_0, v_1)$ and (v_0, v_2) converge to different points, respectively (the dotted line corresponds to $x_C = v_0$).

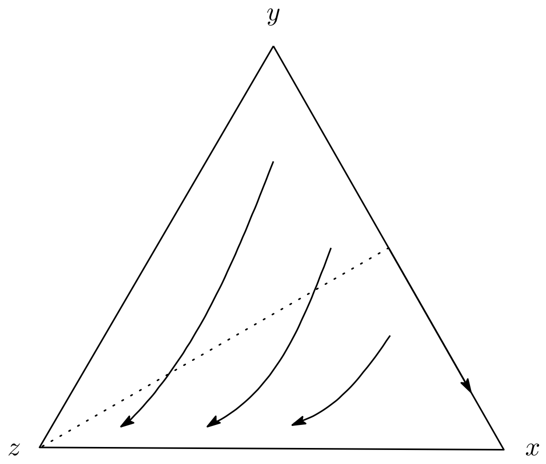

Figure 5: The case where $R + P - T - S < 0$ and $(T-R)/(T-P) \leq r$. In this case, for any $0 < v_0, v_1, v_2 < 1$ ($v_1 \neq v_2$), two initial points $(x_C(0), z(0)) = (v_0, v_1)$ and (v_0, v_2) converge to different points, respectively (the dotted line corresponds to $x_C = v_0$).

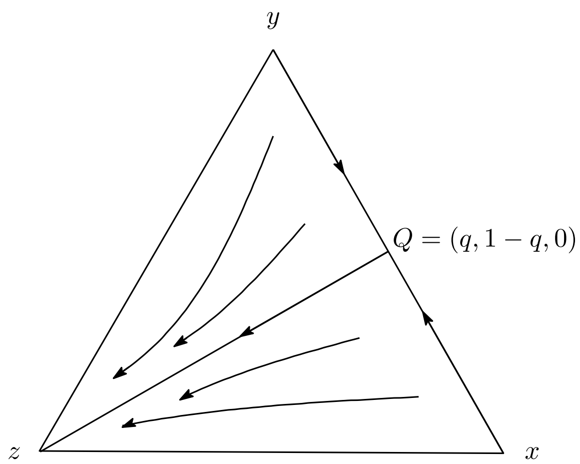

Figure 6: The case where $R + P - T - S < 0$ and $(P - S)/(R - S) < r < (T - R)/(T - P)$. In this case, for $x_C(0) \neq q$ and $0 < z(0) < 1$, any initial point converges to $(x, y, z) = (0, 0, 1)$, and $x_C = x/(x + y)$ converges to q asymptotically. Any initial point with $x_C(0) = q$ and $0 < z(0) < 1$ converges to $(x, y, z) = (0, 0, 1)$ along $x_C = q$.

that an individual cooperates at the same probability as the proportion of cooperation in the population. In addition, we adopt a game between relatives which formulate the fact that an individual is more likely to meet an opponent using the same strategy. These realistic features provide an improved understanding of human behavior. By (2), we describe the fact that with the probability r an individual cooperating at time t meets an opponent cooperating at time t, regardless of always cooperating or using a wait-and-see strategy, and with the probability $1-r$ all individuals are randomly matched. As a result, we show that a wait-and-see strategy survives in almost all conditions for the prisoner's dilemma between relatives.

In particular, we remark on the case where $R+P-T-S < 0$ and $(P-S)/(R-S) < r < (T-R)/(T-P)$. If the available strategies are only cooperation and defection, we obtain a realistic result that both strategies coexist. However, if the available strategies are always cooperating, always defecting and using a wait-and-see strategy, we obtain a more realistic result that all individuals will use a wait-and-see strategy and they will cooperate with the probability q.

Here, we compare the case where we introduce a wait-and-see strategy as the third strategy into the prisoner's dilemma between relatives and the case where we introduce the mixed strategy. We assume that an individual using the mixed strategy cooperates with probability p. Let x, y and w be the proportion of the population always cooperating, always defecting and using the mixed strategy at time t, respectively. Then, we have $x, y, w \geq 0$ and $x + y + w = 1$. From now on, we suppose that $0 < p < 1$ and $0 < w(0) < 1$. Let x_C and x_D be the proportion of the cooperation and defection in the population at time t, respectively. Then, we have $x_C, x_D \geq 0$, $x_C + x_D = 1$, and $x_C = x + pw$. Hence, we have

$$\begin{aligned}\dot{x} &= \{r(R-S) - (P-S) + (1-r)(R+P-T-S)x_C\} \\ &\quad \times x_D x + rp(1-p)(R+P-T-S)wx,\end{aligned}$$

$$\begin{aligned}\dot{y} &= \{(P-S) - r(R-S) - (1-r)(R+P-T-S)x_C\} \\ &\quad \times x_C y + rp(1-p)(R+P-T-S)wy,\end{aligned}$$

$$\begin{aligned}\dot{w} &= \{(P-S) - r(R-S) - (1-r)(R+P-T-S)x_C\} \\ &\quad \times (x_C - p)w - rp(1-p)(R+P-T-S)(1-w)w,\end{aligned}$$

$$\begin{aligned}\dot{x}_C &= \{r(R-S) - (P-S) + (1-r)(R+P-T-S)x_C\} \\ &\quad \times \{xy + (1-p)^2 xw + p^2 yw\} \\ &\quad + rp(1-p)(R+P-T-S)(x_C - p)w.\end{aligned}$$

Consequently, we have the following result.

(1) If $R+P-T-S > 0$, we have $\lim_{t\to\infty} w(t) = 0$, that is, the mixed strategy can not survive regardless of the value of r and p.

(2) If $R+P-T-S<0$, we have $\lim_{t\to\infty} w(t)=0$ when either of the following two conditions are fulfilled;

$$\text{(i) } r<\frac{P-S}{R-S},\ p>\frac{(P-S)-r(T-P)}{r(R+P-T-S)},$$

$$\text{(ii) } r>\frac{T-R}{T-P},\ p<\frac{(T-R)-r(T-P)}{r(R+P-T-S)}.$$

(3) If $R+P-T-S=0$, we have $\lim_{t\to\infty} w(t)=0$ when $r\neq(P-S)/(R-S)$, that is, the mixed strategy can not survive except for $r=(P-S)/(R-S)$.

Therefore, from the viewpoint of survival, our wait-and-see strategy is superior to all the mixed strategies.

Furthermore, we consider a wait-and-see strategy in the "inclusive fitness" approach. Taylor and Nowak [12] analyzed various mechanisms for the emergence of cooperation in the prisoner's dilemma. One of these mechanisms is kin selection, based on the concept of "inclusive fitness". The "inclusive fitness" approach modifies the original payoff matrix A to

$$A'=\begin{bmatrix}(1+r')R & S+r'T\\ T+r'S & (1+r')P\end{bmatrix},$$

where r' is the coefficient of relationship ($0<r'<1$). An individual obtains the sum of his own payoff plus r' times his opponent's payoff. In this framework, all individuals of the population are randomly matched. Hence, we have

$$\begin{aligned}\dot{x}_C=\{&r'(T-P)-(P-S)\\ &+(1+r')(R+P-T-S)x_C\}(1-x_C)x_C.\end{aligned}$$

Consequently, we can see that cooperation can emerge in this game. On the other hand, the calculated thresholds of the personal fitness approach and inclusive fitness approach are different. Moreover, if we introduce a wait-and-see strategy into the game based on the transformed payoff matrix A', then we have

$$\dot{z}=\{(f_{WS}-f_{AC})x+(f_{WS}-f_{AD})y\}z=0.$$

Therefore, we can obtain $z(t)=z(0)$ regardless of the value of r'. This is because the inclusive fitness approach simply modifies payoff matrix. Namely, even if a mechanism for the emergence of cooperation in the prisoner's dilemma transforms the payoff matrix, a wait-and-see strategy can survive in the game based on the transformed payoff matrix as long as we assume that all individuals of the population are randomly matched.

Naturally, there exist many mechanisms for the emergence of cooperation in the prisoner's dilemma. Also, there exist a lot of strategies that vary in time the probability of cooperation in the prisoner's dilemma besides a wait-and-see strategy. Moreover, it is not clear whether a wait-and-see strategy is the best strategy among these strategies. However, our result suggests that the behavior to cooperate at the same probability as the proportion of cooperation in the population is reasonable.

5. The proof of Proposition 1

In this section, we prove Proposition 1.

(0) If $x_C(0)=0$ or 1, we have $x_C(t)=0$ or 1 from (2), respectively. Then, we have $\dot{z}(t)=0$ from (3). Therefore, we have $z(t)=z(0)$.

From now on, we consider the case where $0<x_C(0)<1$.

(1) (i) If $r\le(T-R)/(T-P)$, we have

$$\begin{aligned}\frac{\dot{x}_C(t)}{x_C(t)}&\le\{r(R-S)-(P-S)\\ &\quad+(1-r)(R+P-T-S)x_C(0)\}(1-x_C(0))\\ &=-C_1(<0)\end{aligned}$$

from (2). Since

$$x_C(t)\le x_C(0)\mathrm{e}^{-C_1t}, \tag{4}$$

we have $x_C\to0$, i.e. $x\to0$ as $t\to\infty$. Let $g(t)=z(t)/y(t)$. Then, we have

$$\begin{aligned}\frac{\dot{g}(t)}{g(t)}=\{&r(T-P)-(T-R)\\ &-(R+P-T-S)(1-x_C(t))\}x_C(t).\end{aligned}\tag{5}$$

From (4), we have

$$\begin{aligned}\left|\frac{\dot{g}(t)}{g(t)}\right|&\le\{(T-R)-r(T-P)\\ &\qquad+(R+P-T-S)(1-x_C(t))\}x_C(0)\mathrm{e}^{-C_1t}\\ &\le\{(T-R)-r(T-P)\\ &\qquad+(R+P-T-S)\}x_C(0)\mathrm{e}^{-C_1t}.\end{aligned}$$

Therefore, the right hand side of (5) is integrable in $[0\ \infty)$. Since

$$\begin{aligned}g(t)=g(0)\exp\bigg(\int_0^t&\{r(T-P)-(T-R)\\ &-(R+P-T-S)(1-x_C(s))\}x_C(s)\mathrm{d}s\bigg),\end{aligned}$$

we have

$$\begin{aligned}\lim_{t\to\infty}g(t)=g(0)\exp\bigg(\int_0^\infty&\{r(T-P)-(T-R)\\ &-(R+P-T-S)(1-x_C(s))\}x_C(s)\mathrm{d}s\bigg).\end{aligned}$$

(1) (ii) If $(P-S)/(R-S)\le r$, we can show that $x_D\to0$, i.e. $y\to0$ as $t\to\infty$. Let $h(t)=z(t)/x(t)$. Then, we have

$$\begin{aligned}\frac{\dot{h}(t)}{h(t)}=\{&(P-S)-r(R-S)\\ &-(R+P-T-S)x_C(t)\}(1-x_C(t)).\end{aligned}$$

In the same way as (i), we have

$$\lim_{t\to\infty} h(t) = h(0)\exp\left(\int_0^\infty \{(P-S) - r(R-S) - (R+P-T-S)x_C(s)\}(1-x_C(s))\mathrm{d}s\right).$$

(1) (iii) If $(T-R)/(T-P) < r < (P-S)/(R-S)$ and $x_C(0) < q$, we see that $x_C \to 0$, as $t \to \infty$. Let $g(t) = z(t)/y(t)$. Then, we have

$$\lim_{t\to\infty} g(t) = g(0)\exp\left(\int_0^\infty \{r(T-P) - (T-R) - (R+P-T-S)(1-x_C(s))\}x_C(s)\mathrm{d}s\right)$$

in the same way as (i).

If $x_C(0) > q$, we see that $x_D \to 0$, as $t \to \infty$. Let $h(t) = z(t)/x(t)$. Then, we have

$$\lim_{t\to\infty} h(t) = h(0)\exp\left(\int_0^\infty \{(P-S) - r(R-S) - (R+P-T-S)x_C(s)\}(1-x_C(s))\mathrm{d}s\right)$$

in the same way as (ii).

If $x_C(0) = q$, we can show that $x_C(t) = q$ from (2). Then, from (3), we have

$$\frac{\dot{z}(t)}{z(t)} = -r(R+P-T-S)(1-q)q(1-z(t)).$$

Therefore, we have $z \to 0$ as $t \to \infty$.

(2) (i) If $r \leq (P-S)/(R-S)$, we have $x_C \to 0$, i.e. $x \to 0$ as $t \to \infty$ from (2). Let $g(t) = z(t)/y(t)$. Then, we have

$$\lim_{t\to\infty} g(t) = g(0)\exp\left(\int_0^\infty \{r(T-P) - (T-R) - (R+P-T-S)(1-x_C(s))\}x_C(s)\mathrm{d}s\right).$$

(2) (ii) If $(T-R)/(T-P) \leq r$, we have $x_D \to 0$, i.e. $y \to 0$ as $t \to \infty$. Let $h(t) = z(t)/x(t)$. Then, we have

$$\lim_{t\to\infty} h(t) = h(0)\exp\left(\int_0^\infty \{(P-S) - r(R-S) - (R+P-T-S)x_C(s)\}(1-x_C(s))\mathrm{d}s\right).$$

(2) (iii) If $(P-S)/(R-S) < r < (T-R)/(T-P)$, we can show that $x_C \to q$ from (2). Since

$$\frac{\dot{z}(t)}{z(t)} = -r(R+P-T-S)(1-x_C(t))x_C(t)(1-z(t)),$$

we have $z \to 1$ as $t \to \infty$.

(3) If $R+P-T-S = 0$, we have $\dot{z}(t) = 0$ from (3). Therefore, we have $z(t) = z(0)$.

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