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Numerically repeated support splitting and merging phenomena in a porous media equation with strong absorption

To the memory of my friend Professor Nakaki.

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Abstract. Nonlinear diffusion equations exhibit a wide variety of phenomena in the several fields of fluid dynamics, plasma physics and population dynamics. Among these the interaction between diffusion and absorption suggests a remarkable property in the behavior of the support of solution; that is, after support splitting phenomena appear, the support merges, and thereafter the support splits again. Moreover, *numerically repeated support splitting and merging phenomena* are observed. In this paper, making use of the properties of the particular solutions, we construct an initial function for which such phenomena appear.

Keywords. nonlinear diffusion, support splitting, support merging, finite difference scheme

1. Introduction

The interaction between diffusion and absorption appears in the flow through an absorbing medium occupying all of $\mathbf{R}^1$, and is described in the form of the following initial value problem for the degenerate parabolic equation with an additional lower order term:

$$v_t = (v^m)_{xx} - cv^p, \qquad x \in \mathbf{R}^1, \quad t > 0, \tag{1}$$

$$v(0,x) = v^0(x), \qquad x \in \mathbf{R}^1, \tag{2}$$

where $m > 1$, $0 < p < 1$, c is a positive constant, v denotes the density of the fluid, $-cv^p$ describes volumetric absorption, and $v^0(x) \in C^0(\mathbf{R}^1)$ is nonnegative and has compact support. This equation (1) is called the porous media equation with absorption.

In the behavior of solutions of (1)-(2) there appear *total extinction in finite time* and *finite propagation of the support*, which are caused by the interaction between diffusion and absorption. Moreover, *numerical support splitting and merging phenomena* are observed (see Fig.1).

From analytical points of view, the existence and uniqueness of a weak solution, the total extinction in finite time and the finite propagation of the support are proved by Oleinik, Kalashnikov and Chzou[11], Kalashnikov[5, 6] and Knerr[9].

Rosenau and Kamin[12] tried the numerical computation to (1)-(2) and suggested the possibility of the support to split. Chen, Matano and Mimura[2] constructed the initial function for which the support of the solution splits into multiple connected components in a finite time. From a numerical point of view, Nakaki and Tomoeda[10] constructed the finite difference scheme which realizes such a phenomenon, and obtained the sufficient condition imposed on $v^0(x)$ for which the support splits. Kersner[8] proved the support splitting property in the initial-boundary value problem to (1) by constructing supersolutions.

For *support splitting and merging phenomena*, to the best of our knowledge, we have not been able to find any results. This motivates us to investigate such phenomena. The difficulty is to determine whether or not the support splitting and merging phenomena occur, because the convergence of numerical solutions does not always imply that the exact solution also keeps the same phenomena. In this paper, our approach is based on the properties of the particular solutions of (1) under the following
Assumption A. $m + p = 2$ $(m > 1, 0 < p < 1)$.
Unfortunately, in the case where $m + p \neq 2$ $(m > 1, 0 < p < 1)$, we are unable to construct the explicit solution of (1). This is the reason why we are concerned with the specific case stated in Assumption A.

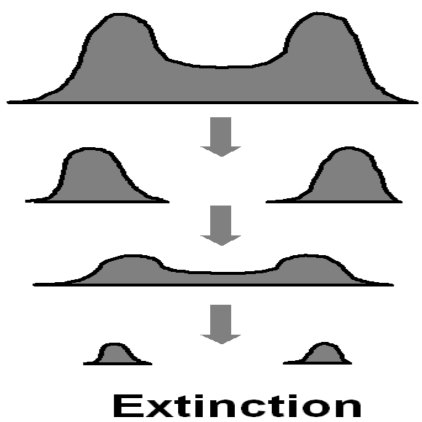


Figure 1: An illustration of support splitting and merging phenomena.

2. Behavior of numerical support

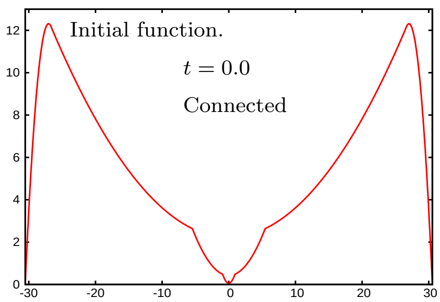
Setting $u = v^{m-1}$, we rewrite (1)–(2) as follows:

$$u_t = muu_{xx} + \frac{m}{m-1}(u_x)^2 - (m-1)c\chi_{\{u>0\}}, \quad (3)$$

$$u(0,x) = u^0(x) \equiv (v^0(x))^{m-1}. \quad (4)$$

We note that the effect of absorption is expressed as the constant $-(m-1)c$ under Assumption A. Our difference scheme approximates the problem (3)–(4) instead of (1)–(2)(see [10]).

Now we show some numerical solutions of u_h^n to (3)-(4) with $m = 1.0625$ and $c = 36$ in the following figures. The convergence of the numerical solutions to the exact one of (1)-(2) is established (see Theorem 3.1 in [10]). *Numerically repeated support splitting and merging phenomena* are observed in the neighbourhood of $x = 0$. This initial function is constructed so that the assumption in Theorem 4 is satisfied, which implies that the occurrence of such phenomena is justified from analytical points of view. In the following we show the construction of it.



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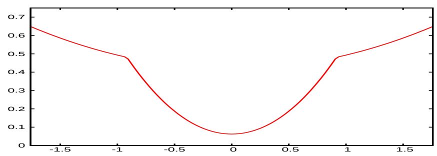

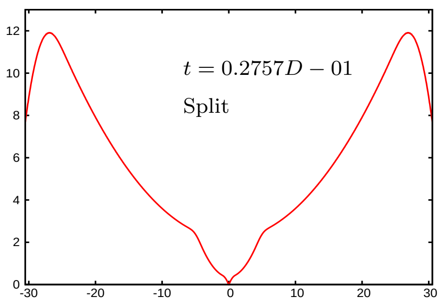

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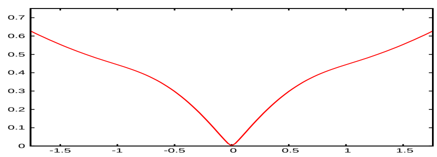

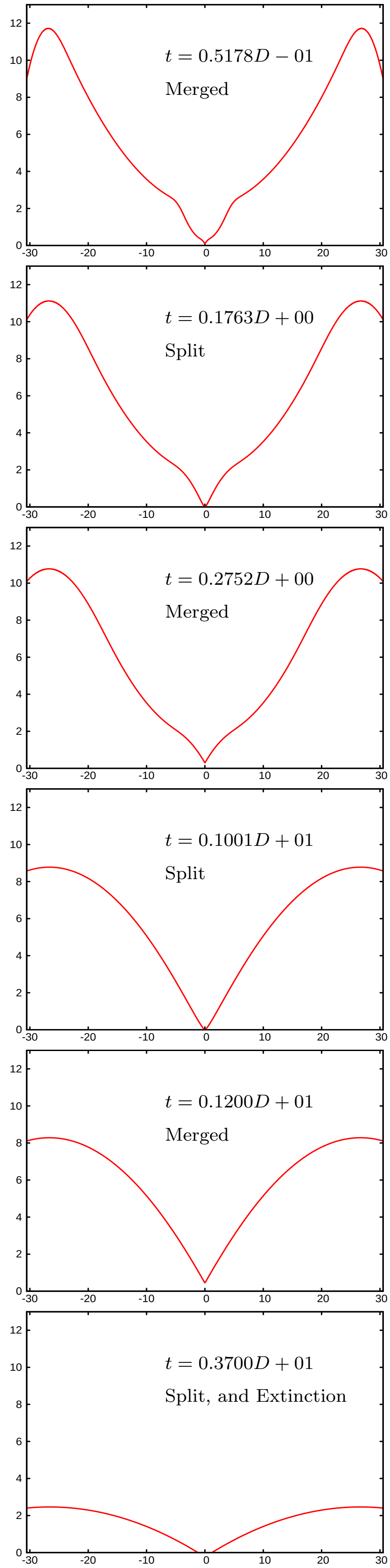

Figure 2: Numerical support splitting and merging phenomena for (3), where $m = 1.0625$ and $c = 36$.

3. Particular solutions and support splitting phenomena

Under Assumption A let us construct Kersner's solution with support expanding property([7]) and Galaktionov and Vazquez's solution with support splitting property([3, 4]) in the form $u(t,x) = f(t) + g(t)h(x)$ to (3). Then it holds that

$$f' + g'h = m(f+gh)gh_{xx} + \frac{m}{m-1}(gh_x)^2 - (m-1)c, \quad (5)$$

where $'$ denotes the partial derivative with respect to t.

Putting $h(x) = -x^2$, we have the system of differential equations

$$\begin{cases} f' = -2mfg - (m-1)c, \\ g' = -\left(2m + \dfrac{4m}{m-1}\right) g^2. \end{cases} \quad (6)$$

Solving (6), we have Kersner's solution with $u(0,x) = [\sigma(1-(x/\rho)^2)]_+$ for two parameters σ (>0) and ρ (>0): (see Fig. 3)

$$u(t,x) = \{A + (2m+4a)t\}^{-1} \times \left[B\{A+(2m+4a)t\}^{\frac{2}{m+1}} - D\{A+(2m+4a)t\}^2 - x^2\right]_+, \quad (7)$$

where $[g]_+ = \max\{g, 0\}$,

$$\begin{cases} A \equiv A(\rho,\sigma) = \frac{\rho^2}{\sigma}, \quad B \equiv B(m,c,\rho,\sigma) = (\sigma + DA)A^{\frac{m-1}{m+1}}, \\ D \equiv D(m,c) = \frac{c}{4a^2}, \quad a = \frac{m}{m-1}. \end{cases} \quad (8)$$

The extinction time $T^*(m,c,\rho,\sigma)(>0)$ and the support $[x_-(t),\ x_+(t)]$ $(0 \le t \le T^*)$ are given by

$$T^*(m,c,\rho,\sigma) = \frac{1}{2m+4a}\left\{\left(\frac{B}{D}\right)^{\frac{m+1}{2m}} - A\right\} \quad (9)$$

and

$$x_\pm(t) = \pm\left\{\left[B\{A+(2m+4a)t\}^{\frac{2}{m+1}} - D\{A+(2m+4a)t\}^2\right]_+\right\}^{\frac{1}{2}}, \quad (10)$$

respectively.

Lemma 1. *$T^*(m,c,\rho,\sigma)$ satisfies*

$$T^*(m,c,\sigma,\rho) \le \frac{\sigma}{(m-1)c} \quad \textit{and}$$

$$\lim_{\rho\to\infty} T^*(m,c,\sigma,\rho) = \frac{\sigma}{(m-1)c} \ (\textit{see Fig. } 3). \quad (11)$$

Putting $h(x) = x^2$ in (5), we similarly obtain Galaktionov and Vazquez's solution with $u(0,x) = \varepsilon x^2 + \hat{\sigma}$ for two parameters $\varepsilon > 0$ and $\hat{\sigma} > 0$:

$$u(t,x) = \{E - (2m+4a)t\}^{-1} \times \left[D\{E-(2m+4a)t\}^2 + G\{E-(2m+4a)t\}^{\frac{2}{m+1}} + x^2\right]_+, \quad (12)$$

where $E \equiv E(\varepsilon) = \varepsilon^{-1}$ and $G \equiv G(m,c,\hat{\sigma},\varepsilon) = (\hat{\sigma} - DE)E^{\frac{m-1}{m+1}}$.

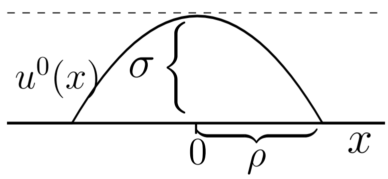

Figure 3: The initial function of Kersner's solution(7).

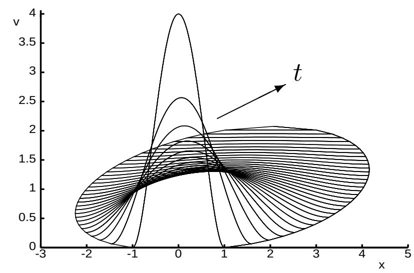

Figure 4: Kersner's solution.

Lemma 2. *G satisfies*

$$\lim_{\varepsilon\to 0} G(m,c,\hat{\sigma},\varepsilon) = -\infty\ . \quad (13)$$

Moreover, in the case where $G(m,c,\hat{\sigma},\varepsilon) < 0$ the following inequalities

$$\frac{\hat{\sigma}}{(m-1)c} < \hat{t}(m,c,\hat{\sigma},\varepsilon) < \hat{T}(m,\varepsilon) \ \textit{and}$$

$$\lim_{\varepsilon\to 0} \hat{t}(m,c,\hat{\sigma},\varepsilon) = \frac{\hat{\sigma}}{(m-1)c} \quad (14)$$

hold and u satisfies (see Fig. 5 and 6)

$$u(t,x) > 0 \left((t,x) \in [0,\ \hat{T}(m,\varepsilon)) \times \mathbf{R}^1 \setminus S\right), \quad (15)$$

$$u(t,x) = 0\,((t,x) \in S), \quad (16)$$

$$\lim_{t\nearrow \hat{T}(m,\varepsilon)} u(t,0) = 0, \quad (17)$$

$$\lim_{t\nearrow \hat{T}(m,\varepsilon)} u(t,x) = \infty \ (x \neq 0), \quad (18)$$

where

$$\hat{t}(m,c,\hat{\sigma},\varepsilon) = \frac{1}{2m+4a}\left\{E - \left(\frac{-G}{D}\right)^{\frac{m+1}{2m}}\right\}, \quad (19)$$

$$\hat{T}(m,\varepsilon) = \frac{E}{2m+4a}, \quad (20)$$

$$S = \left\{ (t,x) \mid t \in \left[\hat{t}(m,c,\hat{\sigma},\varepsilon),\ \hat{T}(m,\varepsilon)\right), \ x^2 \le \{E-(2m+4a)t\}^{\frac{2}{m+1}} \times \left[-G - D\{E-(2m+4a)t\}^{\frac{2m}{m+1}}\right] \right\}. \quad (21)$$

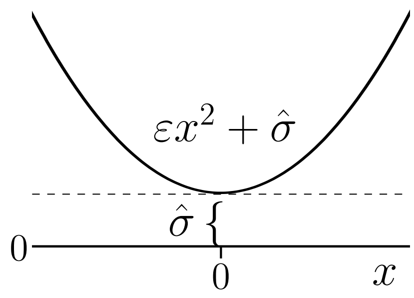

Figure 5: The initial function of Galaktionov and Vazquez's solution (12).

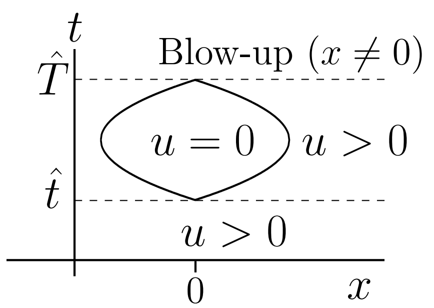

Figure 6: The support of Galaktionov and Vazquez's solution (12).

Lemmas 1 and 2 are shown by simple calculations. In Lemma 2 the appearance of the region S for a sufficiently small ε implies the support splitting phenomena. As a consequence of these lemmas, we obtain

Theorem 1. *For arbitrary numbers σ and $\hat{\sigma}$ $(\sigma > \hat{\sigma} > 0)$ there exist constants $\rho(>0)$ and $\varepsilon(>0)$ such that*

$$0 < \hat{t}(m, c, \hat{\sigma}, \varepsilon) < T^*(m, c, \sigma, \rho). \tag{22}$$

Moreover, for these constants ρ and ε suppose $u^0(x)$ satisfies

$$\left[\sigma\left\{1 - ((x \pm \xi)/\rho)^2\right\}\right]_+ \leq u^0(x) \leq \varepsilon x^2 + \hat{\sigma} \tag{23}$$

for some $\xi(>0)$. Then there exists t' $\left(\hat{t}(m, c, \hat{\sigma}, \varepsilon) < t' < T^*(m, c, \sigma, \rho)\right)$ such that supp $u(t', \cdot)$ *splits into at least two disjoint sets on the interval $[-\xi, \xi]$.*

Remark 1. The last assertion of the theorem is proved by the comparison theorem([1]) which is concerned with the initial functions.

Remark 2. The inequality (22) is satisfied for a sufficiently small number ε and a sufficiently large number ρ.

4. Support splitting and merging phenomena

Using the initial functions of Galaktionov and Vazquez's solution and Kersner's solution, we construct an initial function (4) for which support splitting and merging phenomena appear. Let two functions $\varepsilon x^2 + \hat{\sigma}$ and $[\sigma(1 - ((x \pm \xi)/\rho)^2)]_+$ be tangent to each other at some points $(\pm\hat{x}, \gamma)$ (see Fig. 7).

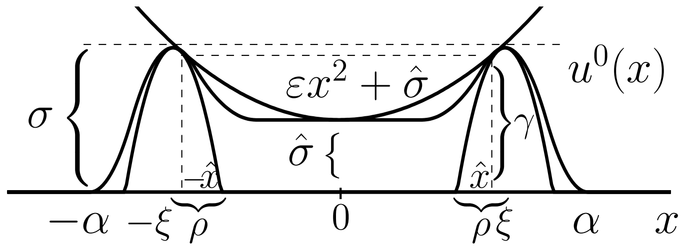

Figure 7: The initial function $u^0(x)$.

Then we have

$$\xi^2 = (\sigma - \hat{\sigma})\left(\frac{1}{\varepsilon} + A\right), \tag{24}$$

$$\left[\sigma\left\{1 - ((x \pm \xi)/\rho)^2\right\}\right]_+ \leq \varepsilon x^2 + \hat{\sigma} \text{ on } \mathbf{R}^1, \tag{25}$$

where $\xi \equiv \xi(m, c, \varepsilon, \hat{\sigma}, \sigma, \rho)$.

From (10) it follows that there exists $t = t^*$ at which $x_\pm^2(t)$ attains the maximum; that is,

$$t^* \equiv t^*(m, c, \sigma, \rho) = \frac{1}{2m + 4a}\left[\left\{\frac{aB}{D(m + 2a)}\right\}^{\frac{m+1}{2m}} - A\right], \tag{26}$$

$$x_\pm^2(t^*) = \frac{(m + a)B}{m + 2a}\left\{\frac{aB}{D(m + 2a)}\right\}^{\frac{1}{m}}. \tag{27}$$

We can immediately state

Theorem 2. *Assume that there exist positive constants $\hat{\sigma}, \varepsilon, \sigma$ $(> \hat{\sigma})$ and ρ such that*

$$0 < \hat{t}(m, c, \hat{\sigma}, \varepsilon) < t^*(m, c, \sigma, \rho), \tag{28}$$

$$\xi^2 < x_\pm^2(t^*(m, c, \sigma, \rho)). \tag{29}$$

Let the initial function (4) satisfy (23). Then there exists t' $\left(\hat{t}(m, c, \hat{\sigma}, \varepsilon) < t' < t^*(m, c, \sigma, \rho)\right)$ such that supp $u(t', \cdot)$ *splits into at least two disjoint sets on the interval $[-\xi, \xi]$. Moreover, $[-\xi,\ \xi] \subset$* supp $u(t^*(m, c, \sigma, \rho), \cdot)$*, which implies that the support is merged on $[-\xi,\ \xi]$.*

In the following let us consider the parameters ε, σ and ρ satisfying (28) for an arbitrarily given number $\hat{\sigma}(>0)$. Let $\sigma(>\hat{\sigma})$ be an arbitrarily fixed number. According to Remark 2 of Theorem 1 we can take ε sufficiently small and ρ sufficiently large so that (22) holds, which implies the occurrence of the support splitting property. However, it seems difficult for us to prove that the second inequality of (28) holds for such a sufficient large number ρ. The reason of the difficulty comes from the following fact:

$$t^*(m, c, \sigma, \rho) = T^*(m, c, \sigma, \rho) + \frac{1}{2m + 4a}\left\{\left(\frac{a}{m + 2a}\right)^{\frac{m+1}{2m}} - 1\right\}\left(\frac{B}{D}\right)^{\frac{m+1}{2m}}, \tag{30}$$

which yields

$$\lim_{\rho \to +\infty} t^*(m, c, \sigma, \rho) = -\infty, \tag{31}$$

where $B \equiv B(m,c,\rho,\sigma)$ is given by (8). The second inequality of (28) may fail. To avoid such a difficulty we have to take ρ and σ sufficiently large so that not only the second inequality of (28) but also (29) hold.

To find the relation between ρ and σ we tried numerical computation in the cases where the ratio $\frac{\rho}{\sigma}$ = constant and where the ratio $\frac{\rho^2}{\sigma}$ = constant. We obtain the good result in the latter case. Then we have

Theorem 3. *For an arbitrary number $\hat{\sigma}(>0)$ there exist $\varepsilon \equiv \varepsilon(m,c,\hat{\sigma})(>0)$ and $\sigma \equiv \sigma(m,c,\hat{\sigma},\varepsilon)(>0)$ satisfying (28) and (29).*

Proof. The first inequality of (28) can be shown by putting $\varepsilon \equiv \varepsilon(m,c,\hat{\sigma}) < \frac{D}{\hat{\sigma}}$. To prove the theorem it suffices to show the second inequality of (28) and (29) for a positive constant σ satisfying the following inequality:

$$\sigma > \left\{\frac{m+1}{m}\left(A+\frac{1}{\varepsilon}\right)\right\}^m D(m+1)A^{1-m} - DA, \qquad (32)$$

where $A \equiv \frac{\rho^2}{\sigma}$ = constant. From (26), (32) and Assumption A we have

$$\begin{aligned} t^* &\equiv t^*(m,c,\sigma,\rho) \\ &= \frac{1}{2m+4a}\left[\left\{\frac{(\sigma+DA)}{D(m+1)}\right\}^{\frac{m+1}{2m}} A^{\frac{m-1}{2m}} - A\right] \\ &> \frac{1}{2m+4a}\left[\left\{\frac{m+1}{m}\left(A+\frac{1}{\varepsilon}\right)\right\}^{\frac{m+1}{2}} A^{\frac{1-m}{2}} - A\right] \\ &= \frac{1}{2m+4a}\left[\left(\frac{m+1}{m}\right)^{\frac{m+1}{2}}\right. \\ &\quad \times \left.\left\{A^{\frac{m+1}{2}} + \frac{m+1}{2\varepsilon}\left(A+\frac{\theta}{\varepsilon}\right)^{\frac{m-1}{2}}\right\} A^{\frac{1-m}{2}} - A\right] \\ &> \frac{1}{2m+4a}\left(\frac{m+1}{m}\right)^{\frac{m+1}{2}} \frac{m+1}{2\varepsilon} \\ &> \frac{1}{\varepsilon(2m+4a)} = \hat{T}(m,\varepsilon) > \hat{t}(m,c,\hat{\sigma},\varepsilon), \end{aligned} \qquad (33)$$

where θ is a positive constant. Thus the second inequality of (28) holds. From (24), (27) and (32) it follows that

$$\begin{aligned} x_\pm^2(t^*) &= \frac{(m+a)(\sigma+DA)A^{\frac{m-1}{m+1}}}{m+2a} \\ &\quad \times \left\{\frac{a(\sigma+DA)}{D(m+2a)}\right\}^{\frac{1}{m}} A^{\frac{m-1}{m(m+1)}} \\ &> \frac{m\sigma}{m+1}\left\{\frac{\sigma+DA}{D(m+1)}\right\}^{\frac{1}{m}} A^{\frac{m-1}{m}} \\ &> \left(A+\frac{1}{\varepsilon}\right) A^{\frac{1-m}{m}} \sigma A^{\frac{m-1}{m}} = \left(A+\frac{1}{\varepsilon}\right)\sigma > \xi^2, \end{aligned} \qquad (34)$$

which implies (29), and the proof is complete. □

The following theorem immediately follows from Theorems 2 and 3.

Theorem 4. *Let $\sigma_0 > 0$ and $N(\geq 2)$ be arbitrary fixed number and integer, respectively. Then we can choose the sequence $\{\varepsilon_{k-1},\ \sigma_k,\ \rho_k\}(k=1,2,\cdots,N)$ satisfying*

$$0 < \hat{t}(m,c,\sigma_{k-1},\varepsilon_{k-1}) < t^*(m,c,\sigma_k,\rho_k), \qquad (35)$$

$$\xi^2(m,c,\varepsilon_{k-1},\sigma_{k-1},\sigma_k,\rho_k) < x_\pm^2(t^*(m,c,\sigma_k,\rho_k)), \qquad (36)$$

where $\rho_k = \sqrt{A\sigma_k}$. Moreover, for the initial function $u^0(x)$ satisfying

$$\left[\sigma_k\left\{1-((x\pm\xi_k)/\rho_k)^2\right\}\right]_+ \leq u^0(x) \leq \varepsilon_{k-1}x^2 + \sigma_{k-1} \quad on\ \mathbf{R}^1 (k=1,2,\cdots,N), \qquad (37)$$

the support splitting and merging phenomena appear at least N times on the interval $[-\xi_1,\xi_1]$, where $\xi_k = \xi(m,c,\varepsilon_{k-1},\sigma_{k-1},\sigma_k,\rho_k)$ $(k=1,2,\cdots,N)$.

We show some numerical parameters such that (35)-(37) hold in the case where $N=3$. Here $\hat{t}_{k-1} = \hat{t}(m,c,\sigma_{k-1},\varepsilon_{k-1})$, $\xi_k = \xi(m,c,\varepsilon_{k-1},\sigma_{k-1},\sigma_k,\rho_k)$, and $t_k^* = t^*(m,c,\sigma_k,\rho_k)$ $(k=1,2,3)$.

Let us explain the parameters in Table 1. For a given constant $\hat{\sigma} = \sigma_0 = 0.063$, we can choose $\varepsilon = \varepsilon_0 = 0.498$ so that the first inequality of (35) is satisfied. Then Galaktionov and Vazquez's solution takes zero at $x=0$ and $t = \hat{t}_0 = 0.029$, and supp $u(\hat{t}_0,\cdot)$ begins to split into at least two disjoint sets. On the other hand, we can choose $\sigma = \sigma_1 = 0.419$ satisfying (32) with $\varepsilon = \varepsilon_0$. The support of Kersner's solution with $\sigma = \sigma_1$ monotonously expands as $t \nearrow t_1^* = 0.080 > \hat{t}_0$, and is given by $[\xi_1 - x_+(t_1^*),\ \xi_1 + x_+(t_1^*)] = [1.035-1.203,\ 1.035+1.203] = [-0.168,\ 2.238]$. Thus supp $u(t_1^*,\cdot) \supset [-\xi_1 - x_+(t_1^*),\ \xi_1 + x_+(t_1^*)]$ holds and the support of u is merged at $t = t_1^*$ on the interval $[-\xi_1,\ \xi_1]$. Similarly, we can choose ε_{i-1}, $\hat{t}_{i-1}$, σ_i, ξ_i, and $t_i^*(i=2,3)$, which implies that the support splitting and merging property appears at least 3 times on $[-\xi_1,\ \xi_1]$. We show the initial function $u^0(x)$ in Figures 8 and 9.

Table 1: Numerical examples related to Theorem 4.

$m = 1.0625,\quad c = 36.0,\quad A = 1$						
$\sigma_0 = 0.063$					$\varepsilon_0 = 0.498$	$\hat{t}_0 = 0.029$
$\sigma_1 = 0.419$	$t_1^* = 0.080$	$x_+(t_1^*) = 1.203$	$\xi_1 = 1.035$	$\rho_1 = 0.647$	$\varepsilon_1 = 0.074$	$\hat{t}_1 = 0.192$
$\sigma_2 = 2.219$	$t_2^* = 0.436$	$x_+(t_2^*) = 5.738$	$\xi_2 = 5.099$	$\rho_2 = 1.489$	$\varepsilon_2 = 0.014$	$\hat{t}_2 = 1.015$
$\sigma_3 = 12.31$	$t_3^* = 2.333$	$x_+(t_3^*) = 29.94$	$\xi_3 = 27.04$	$\rho_3 = 3.509$		

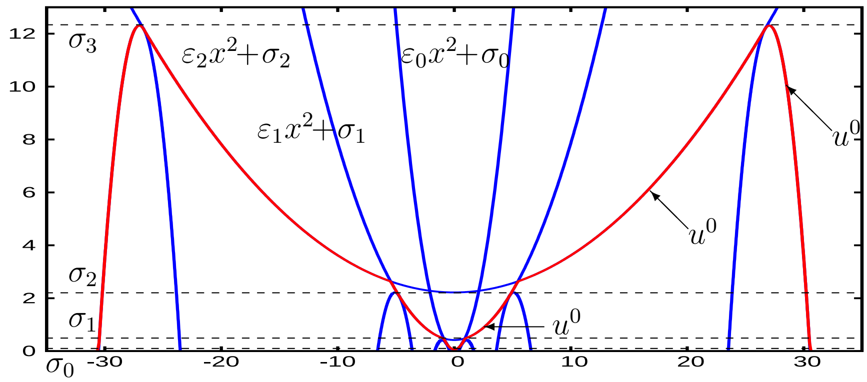

Figure 8: The initial functions of Galaktionov and Vazquez's solution and Kersner's solution, and $u^0(x)$, where $m = 1.0625$ and $c = 36$.

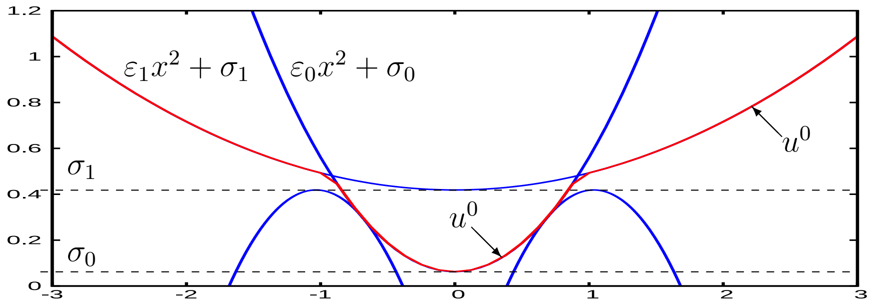

Figure 9: A close up of Figure 8.

5. Numerical support in the initial-boundary value problem

We show the numerical behavior of the support of the solutions of (3) with the conditions:

$$u(0,x) = u^0(x), \qquad x \in (-1.5,\ 1.5), \tag{38}$$

$$u(t,\pm 1.5) = \varphi(t), \qquad t > 0. \tag{39}$$

Put $m = 1.5$ and $c = 6$. We consider the following three cases.

Case (I). $u^0(x) = 1.5$ and $\varphi(t) = 1.5$;
Case (II). $u^0(x) = 2.0$ and $\varphi(t) = 1.5 + 0.5\cos(2\pi t)$;
Case (III). $u^0(x) = 2.0$ and $\varphi(t) = 1.5 + 0.5\cos(12\pi t)$.

In Case (I) the numerical solution converges to the solution $\bar{u}(x) = |x|(|x| \le 1.5)$ as $t \to \infty$, which is derived from $a(\bar{u}_x)^2 - (m-1)c = 0$ on the right side of (3). If $0 \le u^0(x) < \bar{u}(x)$ and $\varphi(t) < 1.5$, the comparison theorem yields the appearance of the support splitting phenomena. If $u^0(x) > 1.5$ and $\varphi(t) > 1.5$, the support never splits.

In Case (II) numerically repeated support splitting and merging phenomena are observed (see Figure 10). The boundary value $\varphi(t)$ with the period 1 takes the maximum 2.0 and the minimum 1.0. On the other hand, the period is $\frac{1}{6}$ in Case (III) and is less than that in Case (II). In this case the support splitting phenomena are not observed (see Figure.11). The numerical computation suggests that the appearance of the support splitting and merging phenomena depends on the period of $\varphi(t)$. So, the mathematical analysis for such phenomena is needed.

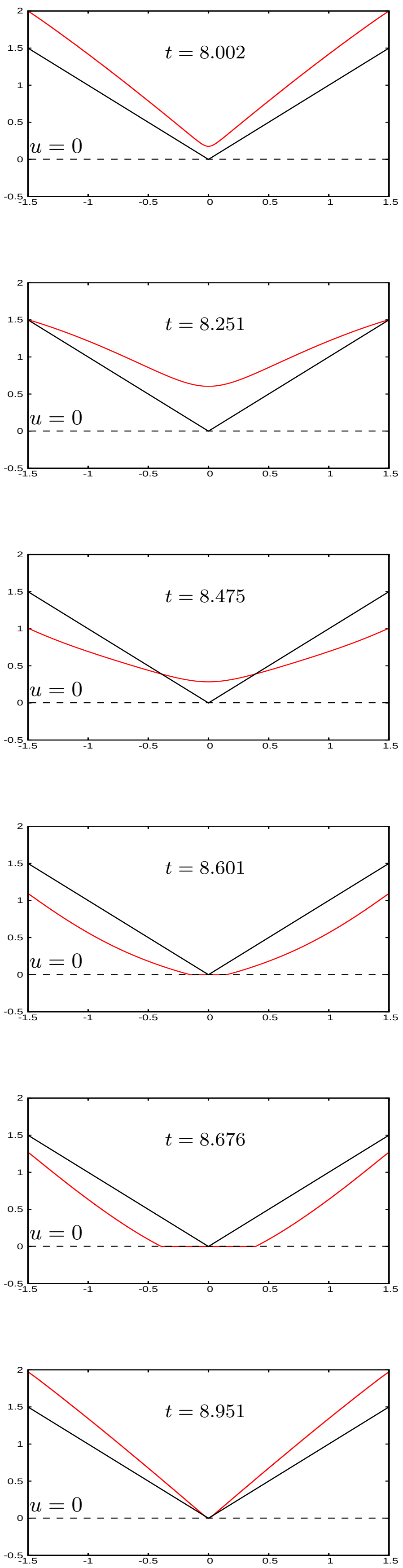

Figure 10: Numerically repeated support splitting and merging phenomena in Case (II), where $m = 1.5$ and $c = 6$.

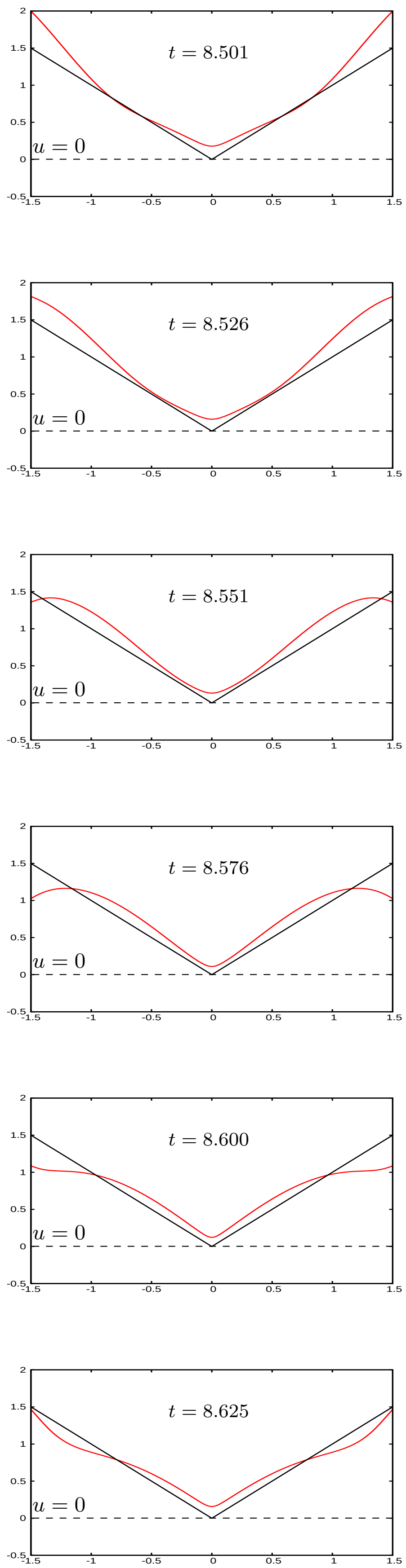

Figure 11: Numerical support no-splitting phenomena in Case (III), where $m = 1.5$ and $c = 6$.

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