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An inverse problem of thickness design for single layer textile material under low temperature

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Abstract. Based on the model of steady-state heat and moisture transfer through textiles, we propose an inverse problem of thickness design for single layer textile material under low temperature. Adopting the idea of regularization method, solving the inverse problem can be formulated into a function minimization problem. Combining the finite difference method for ordinary differential equations with direct search method of one-dimensional minimization problems, we derive three kinds of iteration algorithms of regularized solution for the inverse thickness problem. Numerical simulation is achieved in order to verify the validity of proposed methods.

Keywords. textiles; heat and moisture transfer; inverse problems; thickness design; regularization method; numerical solution

1. Introduction

Simultaneous heat and moisture transfer in porous media is of growing interest in a wide range of science and engineering fields, such as civil engineering, safety analysis of dam, meteorology, energy storage and energy conservation, functional clothing design. In these applications, modeling becomes interesting important since it provides an efficient way for evaluating new designs or testing new materials.

In the recent ten years, some researchers, such as Jintu Fan, Yi Li and so on, have already put forward a lot of dynamic models of coupled heat and moisture transfer through porous clothing assemblies and fibrous insulation. The models concerning with temperature and moisture fields in fibrous insulation, from the knowledge of initial and boundary conditions, constitutes direct problems of heat and mass transfer [1-4]. Based on these models, they have designed numerical methods to solve the direct problems, such as finite difference method, finite volume method, control volume-time domain recursive method [5], and the numerical results are well matched with experimental results.

All these models have given predictions on the properties of heat and moisture transfer through different textiles and have shown effective in clothing design. But to our knowledge, we have not seen the mathematical formulation of inverse problems of textile materials design on heat and moisture transfer properties. Therefore, in this paper, we propose the formulation of an inverse problem of thickness design, which is based on the steady-state model of coupled heat and moisture transfer through parallel pore textiles [6].

The inverse problem of thickness design for textiles is of highly theoretical advantages since it can predict and guide the textile design and clothing equipment design sci-entifically. Meanwhile it has practical significance in the development of advanced textile material and protecting human health in harsh environment. It's necessary that we should study numerical methods of the inverse problem mathematically.

Generally speaking, inverse problems are mathematically classified as ill-posed, that is, their solutions may not satisfy the requirements of existence, uniqueness and stability under small perturbations in the input data. Despite the ill-posed characteristics, the solution of an inverse problem can be obtained through its reformulation in terms of a well-posed problem, such as a minimization problem associated with some kind of regularization/stabilization technique. Different methods based on such an approach have been successfully used to derive estimation of parameters and functions, in linear and non-linear inverse problems [7].

This paper is composed of the following sections. In Section 2, we will introduce a mathematical model of steady-state coupled heat and moisture transfer through parallel pore textiles, and furthermore put forward an inverse problem of thickness design for the first time. In Section 3, according to the idea of regularization methods, we establish iteration schemes to numerically solve the inverse problem. In Section 4, we give an algorithm to numerically solve the direct problem. In Section 5, we make numerical simulation under two different low temperature conditions and obtain the thickness of three different materials. The numerical simulation show validity of the algorithm and presentation of the inverse design problem.

2. Mathematical Formulation of an Inverse Problem of Thickness Design

Textile material design is a kind of inverse problems in mathematical physics fields. In this paper, we consider an inverse problem of thickness design based on the steady-state model of heat and moisture transfer through parallel pore textiles under low temperature. The model of steady-state heat and moisture transfer through parallel pore textiles can be described as a mixed problem of coupled ordinary differential equations[6]:

$$(1) \qquad \frac{k_1 \varepsilon(x) r(x)}{\tau(x)} \cdot \frac{p_v}{T^{3/2}} \cdot \frac{dp_v}{dx} + m_v(x) = 0$$

$$(2) \qquad \frac{dm_v}{dx} + \Gamma(x) = 0$$

$$(3) \qquad \kappa \frac{d^2 T}{dx^2} + \lambda \Gamma(x) = 0$$

$$(4) \qquad \Gamma(x) = \frac{-k_2 \varepsilon(x) r(x)}{\tau(x)} \cdot (p_{sat} - p_v) \cdot \frac{1}{\sqrt{T}}$$

and

$$(5) \qquad \begin{cases} T(0) = T_L \\ T(L) = T_R \\ m_v(0) = m_{v,0} \\ p_v(0) = p_{v,0} \end{cases}$$

where $0 < x < L$, L represents the thickness of textile material. T is temperature(K); $m_v(x)$ is mass flux of water vapor$(kg/m^2 \cdot s)$; p_v is water vapor pressure(pa); $\Gamma(x)$ is the rate of condensation$(kg/m^3 \cdot s)$. The saturation vapor pressure within the parallel pore is given as follows[8]:

$$(6) \qquad p_{sat}(T) = 100 \cdot exp[18.956 - \frac{4030}{(T - 273.16) + 235}]$$

k_1 and k_2 are both constants which are related with molecular weight and gas constant; $\varepsilon(x)$ is porosity of textile surface; $r(x)$ is radius of cylindrical pore(m); $\tau(x)$ is effective tortuosity of the textile; λ is latent heat of sorption and condensation of water vapor(J/kg); κ is thermal conductivity of textiles$(W/m \cdot K)$; Let $k_3 = \frac{\kappa}{\lambda}$.

$T(0)$ and $T(L)$ are the temperatures of inner fabric and outside fabric respectively; $m_{v,0}$ is mass flux of water vapor of inner side of fabric; $p_{v,0}$ is water vapor pressure of inner side of fabric.

The above mixed problem (1)-(5) of coupled ordinary differential equations is usually called a direct problem (**DP: direct problem**).

Next, we consider an inverse problem of thickness design for single layer textile material.

Suppose that the environmental temperature and relative humidity are given as follows:

$$(T, RH) \in [T_{min}, T_{max}] \times [H_{min}, H_{max}],$$

where T_{min} and T_{max} are minimum average temperature and maximum average temperature at a specific place and during a specific time period respectively; Similarly H_{min} and H_{max} are minimum average relative humidity and maximum average relative humidity respectively.

Suppose that the structure and type of single layer textile are known. The structure of textile includes the radius of pore, porosity of textile surface and effective tortuosity of the textile.

The literatures on clothing thermal comfort have indicated that the comfort indexes in the clothing microclimate, which is located between the skin surface and the inner surface of fabric, are given as follows [8]: temperature $(32 \pm 1)^o C$, relative humidity $(50\% \pm 10\%)$, wind speed (25 ± 15)cm/s.

According to the requirements of clothing thermal comfort, we intend to determine the fabric thickness L. Thus, the inverse problem of thickness design can be formulated as follows:

IP(inverse problem): Given the environmental temperature and relative humidity and the above comfort indexes, according to the boundary value conditions

$$(7) \qquad \begin{cases} T(0) = T_L \\ T(L) = T_R \\ m_v(0) = m_{v,0} \\ p_v(L) = p_{v,R}, \end{cases}$$

we need determine the thickness L of fabric through the model of ODEs (1)-(4), where $p_{v,R}$ is related with the temperature and relative humidity of environment.

3. Numerical Algorithms of the Inverse Problem

With regard to the inverse problem of thickness design for textiles, we construct numerical algorithm to implement numerical simulation.

3.1. Regularized Solution of the Inverse Problem

In order to obtain the regularized solution, we discretize the combination of environmental temperature and humidity as (T_i, RH_j) $(i = 1, 2, \cdots, k; j = 1, 2, \cdots, m)$. Let $RH_{i,j,0}(x)$ is relative humidity of inner fabric, which will be solved by coupled ordinary differential equations. Suppose that RH_0^* is experience value of relative humidity in comfortable state.

We can attribute the inverse problem to the following least squared problem:

$$min \sum_{i=1}^{k} \sum_{j=1}^{m} (RH_{i,j,0}(x) - RH_0^*)^2$$

Since above least squared problem doesn't exist unique solution or the solutions are unstable, we use regularized idea to improve the least squared method.

In this respect, we define the following function:

$$J(x) = \alpha \cdot x^2 + \sum_{i=1}^{k}\sum_{j=1}^{m}(RH_{i,j,0}(x) - RH_0^*)^2$$

This function is different from the least squared function, as it is added a penalty term on the least squared function, where $\alpha > 0$ is a regularization parameter. Set $M = [0, L]$, which is called the permissible solution set. If x_{reg} satisfies

$$J(x_{reg}) = min_{x \in M}J(x)$$

then it is called the regularized solution of the inverse problem, or the generalized solution.

3.2. Iteration Algorithms of the Regularized Solution

According to the idea that direct problem solving is combined with search iteration of one-dimension minimization problem, we construct the iteration algorithm:

$$x_{n+1} = x_n + \Delta_n \cdot d_n, n = 1, 2, \cdots$$

to solve the minimization problem such that $J(x_{n+1}) < J(x_n)$, where x_1 is arbitrarily given, and x_n satisfies:

$$(8) \begin{cases} \frac{k_1\varepsilon(x)r(x)}{\tau(x)} \cdot \frac{p_v}{T^{3/2}} \cdot \frac{dp_v}{dx} + m_v(x) = 0, 0 < x < x_n \\ \frac{dm_v}{dx} + \Gamma(x) = 0, 0 < x < x_n \\ \kappa\frac{d^2T}{dx^2} + \lambda\Gamma(x) = 0, 0 < x < x_n \\ \Gamma(x) = \frac{-k_2\varepsilon(x)r(x)}{\tau(x)} \cdot (p_{sat} - p_v) \cdot \frac{1}{\sqrt{T}}, 0 < x < x_n \end{cases}$$

$$(9) \begin{cases} T(0) = T_L \\ T(x_n) = T_R \\ m_v(0) = m_{v,0} \\ p_v(x_n) = p_{v,R}, \end{cases}$$

where the search step Δ_n, and the search direction d_n can be determined by search method of one-dimensional minimization problems.

3.3. Search Method of One-Dimensional Minimization Problems

The optimization problem involved in this paper is a single variable problem. As we know, $RH_{i,j,0}(x)$ is relative humidity of inner fabric which is a numerical solution calculated by coupled ordinary differential equations, and it is difficult to obtain the derivative of $RH_{i,j,0}(x)$, hence we must use the direct search method. Taking this actual situation into account, we use Hooke-Jeeves pattern search algorithm[9-10], direct search algorithm by Cai[11] and 0.618 method [12] respectively to solve the above optimization problem.

3.3.1. The Hooke-Jeeves Pattern Search Algorithm

Step 1. x_1 is given. Set initial step $\Delta_1 > 0$, acceleration factor $\gamma \geq 1$, reduced rate $\beta \in (0,1)$, permissible error $\varepsilon > 0$, search direction $e_1 = 1, e_2 = -1$. set $y_1 = x_1, i = 1$.

Step 2. If $J(y_1 + \Delta_1 \cdot e_1) < J(y_1)$, then

$$y_2 = y_1 + \Delta_1 \cdot e_1$$

carry out the step 4; otherwise, carry out the step 3.

Step3. If $J(y_1 + \Delta_1 \cdot e_2) < J(y_1)$, then

$$y_2 = y_1 + \Delta_1 \cdot e_2$$

carry out the step 4; otherwise, if $J(y_1 + \Delta_1 \cdot e_2) \geq J(y_1)$, then

$$y_2 = y_1,$$

carry out the step 4.

Step 4. If $J(y_2) < J(x_i)$, carry out the step 5; otherwise, if $J(y_2) \geq J(x_i)$, carry out the step 6.

Step 5. $x_{i+1} = x_i; y_1 = x_{i+1} + \gamma(x_{i+1} - x_i); i = i + 1$; go to the step 2.

Step 6. If $\Delta_1 \leq \varepsilon$, then stop,$x^* = x_i$; otherwise, $\Delta_1 = \beta \cdot \Delta_1; y_1 = x_i; x_{i+1} = x_i; i = i + 1$; go to the step 2.

3.3.2. Direct search algorithm proposed by Cai

Step 1. $\eta \in [\frac{1}{2}, 1)$ is given. Set initial values $x_1^0, x_2^0 \in [0, L], x_1^0 \neq x_2^0$. Suppose $J(x_1^0) < J(x_2^0)$ (otherwise, exchange x_1^0 and x_2^0), $k = 0$.

Step 2. If $\frac{|x_1^k - x_2^k|}{1 - \eta} \geq L$, continue; otherwise, choose the initial values again.

Step 3. If $|x_1^k - x_2^k| \leq \varepsilon$, go to the step 9.

Step 4. $x_*^k = x_1^k - \eta(x_2^k - x_1^k)$

Step 5. If $J(x_*^k) \leq J(x_1^k)$ and $x_*^k \in [0, L]$, then $x_1^{k+1} = x_*^k; x_2^{k+1} = x_1^k; k = k + 1$; go to the step 3.

Step 6. If $J(x_*^k) > J(x_1^k)$ or $x_*^k \notin [0, L]$, then $x_{**}^k = x_1^k + \eta(x_2^k - x_1^k)$.

Step 7. If $J(x_{**}^k) < J(x_1^k)$, then $x_1^{k+1} = x_{**}^k; x_2^{k+1} = x_1^k; k = k + 1$, go to the step 3.

Step 8. If $J(x_1^k) \leq J(x_{**}^k) < J(x_2^k)$, then $x_1^{k+1} = x_1^k; x_2^{k+1} = x_{**}^k; k = k + 1$, go to the step 3.

Step 9. $x^* = x_1^k$, stops.

3.3.3. 0.618 Method

Step 1. An initial interval $[a_1, b_1]$ is given. Set permissible error $\varepsilon > 0$. Choose the explosive point λ_1 and μ_1, calculate the function values $J(\lambda_1)$ and $J(\mu_1)$:

$$\lambda_1 = a_1 + 0.382(b_1 - a_1), \mu_1 = a_1 + 0.618(b_1 - a_1).$$

$k = 1$.

Step 2. If $b_k - a_k < \varepsilon$, then stop,$x^* = \frac{1}{2}(a_k + b_k)$; otherwise, if $J(\lambda_k) > J(\mu_k)$, go to the step 3; if $J(\lambda_k) \leq J(\mu_k)$, go to the step 4.

Step 3. $a_{k+1} = \lambda_k, b_{k+1} = b_k, \lambda_{k+1} = \mu_k,$

$$\mu_{k+1} = a_{k+1} + 0.618(b_{k+1} - a_{k+1}),$$

calculate $J(\mu_{k+1})$ go to the step 5.

Step 4. $a_{k+1} = a_k, b_{k+1} = \mu_k, \mu_{k+1} = \lambda_k,$

$$\lambda_{k+1} = a_{k+1} + 0.618(b_{k+1} - a_{k+1}),$$

calculate $J(\lambda_{k+1})$, go to the step 5.

Step 5. $k = k + 1$,go to the step 2.

4. Numerical Computation of the Direct Problem

We decouple the ordinary differential equations (8) to obtain the following two points boundary value problem of nonlinear integro-differential equation:

$$\sqrt{T}T'' = \frac{k_2}{k_3} \cdot A(x)\{p_{sat}(T(x)) -$$

$$\sqrt{p_v^2(x_n) + 2\int_x^{x_n} \frac{1}{k_1 \cdot A(s)} \cdot T^{3/2}(s) \cdot [k_3 \cdot T'(s) + C_1]ds\}}$$

$$T(0) = T_L, T(x_n) = T_R$$

.

Subsequently

$$p_v(x) = \sqrt{p_v^2(x_n) + 2\int_x^{x_n} \frac{1}{k_1 \cdot A(s)} T^{3/2}(s)[k_3 \cdot T'(s) + C_1]ds}.$$

where $C_1 = m_v(0) - k_3 \cdot T'(0)$,

Now, we discretize the above differential equations by means of the finite difference method, and obtain following difference equation:

$$\sqrt{T_i} \cdot \frac{T_i - 2T_{i-1} + T_{i-2}}{h^2}$$

$$= \frac{k_2}{k_3} A(x_i)\{p_{sat}(T_i) -$$

$$\sqrt{p_v^2(x_n) + 2\sum_{j=i}^{N-1} \frac{1}{k_1 A(x_{j+1})} T_{j+1}^{3/2}[k_3 \frac{T_{j+1} - T_j}{h} + C_1^*] \cdot h\}}$$

$$i = 2, \cdots, N-1$$

$$\sqrt{T_N} \cdot \frac{T_N - 2T_{N-1} + T_{N-2}}{h^2} = \frac{k_2}{k_3} A(x_N) [p_{sat}(T_N) - p_v(x_n)]$$

$$p_{v,i} = \sqrt{p_{v,R}^2 + 2\sum_{j=i}^{N-1} \frac{1}{k_1 A(x_{j+1})} T_{j+1}^{3/2}[k_3 \frac{T_{j+1} - T_j}{h} + C_1^*] \cdot h}$$

$$i = 0, 1, 2, \cdots, N-1$$

where $h = \frac{x_n}{N}, C_1^* = m_{v,0} - k_3 \cdot \frac{T_1 - T_0}{h}$.

As we know, T_0 and T_N are both known, we can use interpolation method to obtain the approximation value of T_1 and T_{N-1}. Thus, we obtain the explicit difference scheme on variable $T_{N-2}, \cdots, T_2$. Subsequently

$$p_{v,0} = \sqrt{p_{v,R}^2 + 2\sum_{j=0}^{N-1} \frac{1}{k_1 A(x_{j+1})} T_{j+1}^{3/2} \frac{T_{j+1} - T_j}{h} + C_1^*]\frac{x_n}{N}}$$

$$RH_{i,j,0}(x_n) = \frac{p_{v,0}}{p_{sat}(T_0)}$$

$$= \frac{\sqrt{p_{v,R}^2 + 2\sum_{j=0}^{N-1} \frac{1}{k_1 A(x_{j+1})} T_{j+1}^{3/2}[k_3 \frac{T_{j+1} - T_j}{h} + C_1^*]\frac{x_n}{N}}}{100 \cdot exp[18.956 - \frac{4030}{(T_0 - 273.16) + 235}]}$$

After solving the above direct problem, we get the numerical solution of relative humidity of inner fabric $RH_{i,j,0}(x_n)$, but in general, its value doesn't belong to the comfort index value interval.

5. Numerical solutions

In this section, numerical simulation is carried out to verify the validity of above numerical method. We suppose that the initial mass flux of water vapor is $m_v(0) = 3.3084 \times 10^{-5} kg/m^2 \cdot s$. The temperature of the inner side of fabric is assumed to be 32^oC to guarantee that temperature in microclimate is in the comfort index interval, that is $T(0) = 305.16K$. In the model, $k_1 = 0.00006$, $k_2 = 0.00007$.

5.1. Structure and Type of Materials

We take wool, polyester and polypropylene as examples to implement simulation respectively. Structure and type parameters of all three textile materials are listed in Table 1.

Table 1: Structure and type parameters of textile materials

Material	Wool	Polyester	Polypropylene
Radius(m)	1.0×10^{-5}	1.0×10^{-5}	1.0×10^{-5}
Porosity	0.915	0.88	0.87
Thermal Conductivity	0.052	0.084	0.071
Effective tortuosity	1.2	1.2	1.2

Besides, the latent heat of sorption or condensation of water vapor is determined by moisture level in the fabrics, and it has nothing to do with the type of material, we choose $\lambda = 2260 \times 10^3 J/kg$ in wet region for three different materials.

We assume that thermal conductivity of textiles is a constant due to small changes in the environment: $T \in [-10^oC, 10^oC]$, $RH \in [30\%, 90\%]$.

5.2. Measurements on Environmental Temperature and relative Humidity

We choose two different environmental conditions under low temperature for simulation. Firstly, we make isometry

subdivision on the interval $[T_{min}, T_{max}]$ and $[H_{min}, H_{max}]$ respectively , where $[T_{min}, T_{max}]$ is decomposed into k equal portions, and $[H_{min}, H_{max}]$ is decomposed into m equal portions. In simulation, we choose $k = 10$, $m = 10$.

Example 1.

Environmental conditions: $T_{min} = -10^oC, T_{max} = 0^oC$, $H_{min} = 40\%, H_{max} = 90\%$.

Example 2.

Environmental conditions:$T_{min} = 0^oC, T_{max} = 10^oC$, $H_{min} = 30\%, H_{max} = 85\%$.

5.3. GRAPHS OF FUNCTION $J(x)$

Let's study the property of function $J(x)$ for different materials and environmental conditions through the following function graphs.

The graphs of $J(x)$ for above different materials in Example 1 are shown in Fig 1-3.
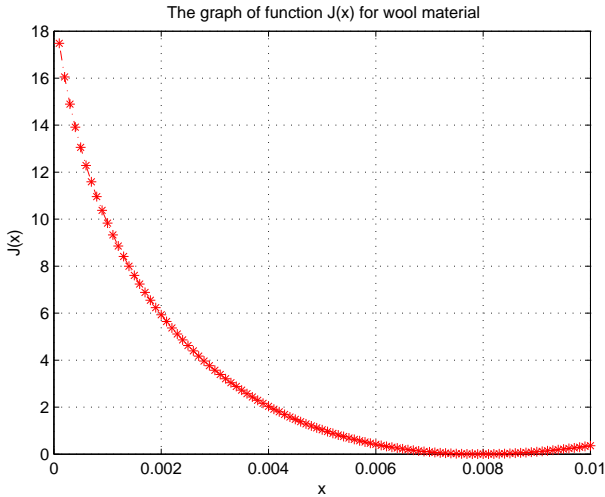

Figure 2: The graph of function J(x) for polyester in Example 1

Figure 1: The graph of function J(x) for wool in Example 1

The graphs of $J(x)$ for above different materials in Example 2 are shown in Fig 4-6.

From above six graphs, it's easy to see that the minimum point of function $J(x)$ is unique in the interval $[0, 0.01]$. This property illustrates that the regularized solution of the inverse problem of textile thickness design is also unique.

5.4. THICKNESS DETERMINATION OF TEXTILE MATERIAL

Using three algorithms in section 3, we can obtain the numerical solutions of thickness for wool, polyester and polypropylene respectively in Example 1 and Example 2.
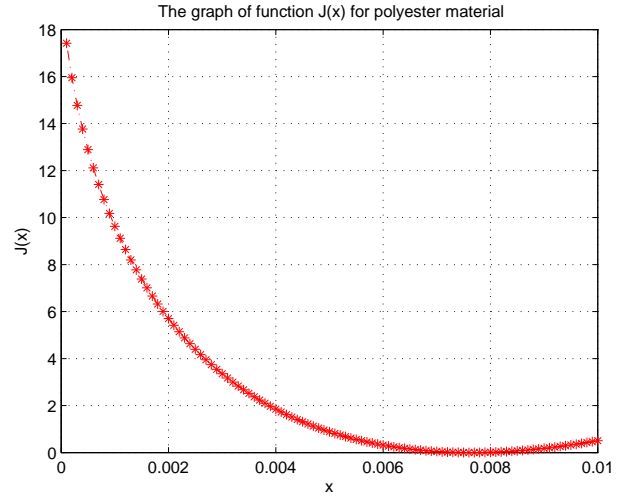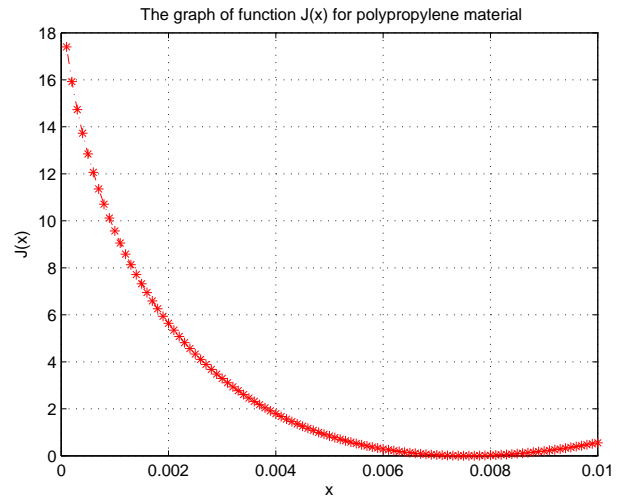

Figure 3: The graph of function J(x) for polypropylene in Example 1

Figure 4: The graph of function J(x) for wool in Example 2

Figure 5: The graph of function J(x) for polyester in Example 2

Figure 6: The graph of function J(x) for polypropylene in Example 2

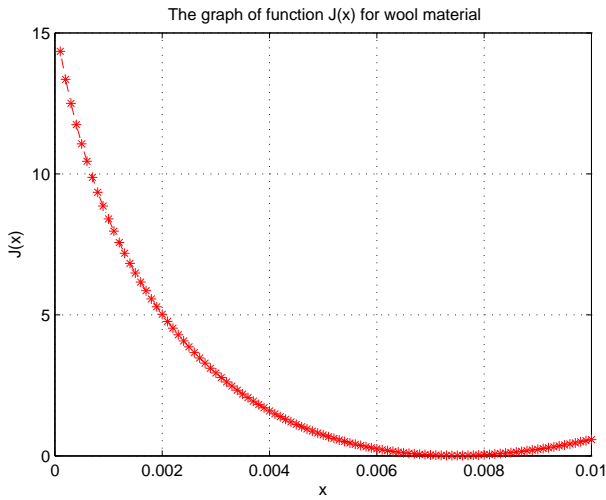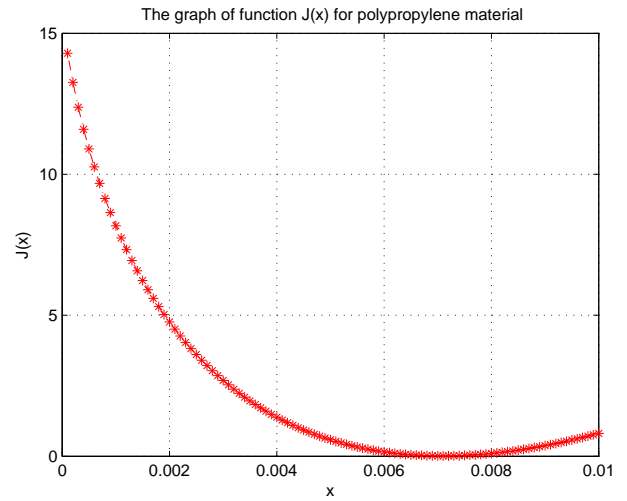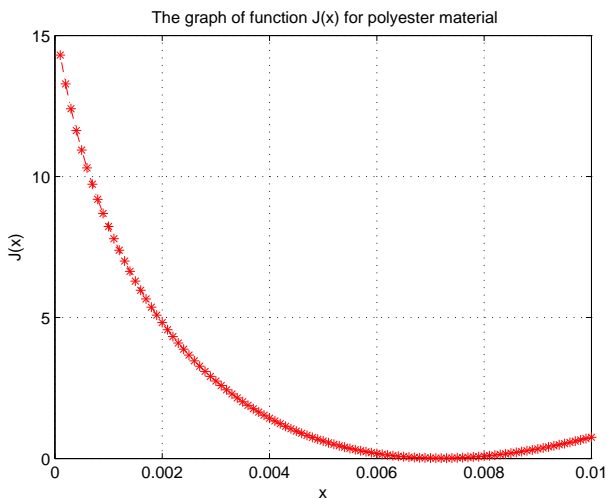### 5.4.1. HOOKE-JEEVES PATTERN SEARCH ALGORITHM

Set acceleration factor $\gamma = 3$, reduced rate $\beta = 0.5$, initial step $\Delta_1 = 10^{-4}$, regularization parameter $\alpha = 0.3$, permissible error $\varepsilon = 10^{-5}$.

5.4.2. CAI'S DIRECT SEARCH ALGORITHM

Set initial values $x_1^0 = 0.0002, x_2^0 = 0.01, \eta = 0.5$, error accuracy $\varepsilon = 10^{-5}$, regularization parameter $\alpha = 0.3$.

5.4.3. 0.618 METHOD

Set initial interval $[a_1, b_1] = [0.0002, 0.01]$, error accuracy $\varepsilon = 10^{-5}$, regularization parameter $\alpha = 0.3$.

Numerical results of thickness designed for wool, polyester and polypropylene in Example 1 are shown in the table 2-4.

Table 2. Numerical results of thickness design for wool in Example 1

Algorithms	H-J	Cai's method	0.618
Initial values	Thickness	Thickness	Thickness
(m)	(cm)	(cm)	(cm)
0.0005	0.7875		
0.001	0.7888	0.7952	0.7951
0.007	0.7938		
0.01	0.795		

Table 3. Numerical results of thickness design for polyester in Example 1

Algorithms	H-J	Cai's method	0.618
Initial values (m)	Thickness (cm)	Thickness (cm)	Thickness (cm)
0.0005	0.765		
0.001	0.765	0.7646	0.7645
0.007	0.765		
0.01	0.7588		

Table 4. Numerical results of thickness design for polypropylene in Example 1

Algorithms	H-J	Cai's method	0.618
Initial values (m)	Thickness (cm)	Thickness (cm)	Thickness (cm)
0.0005	0.75625		
0.001	0.75625	0.7560	0.7557
0.007	0.75625		
0.01	0.75625		

Numerical results of thickness design for wool, polyester and polypropylene in Example 2 are shown in the table 5-7.

Table 5. Numerical results of thickness design for wool in Example 2

Algorithms	H-J	Cai's method	0.618
Initial values (m)	Thickness (cm)	Thickness (cm)	Thickness (cm)
0.0005	0.7525		
0.001	0.74625	0.7454	0.7455
0.007	0.74625		
0.01	0.74625		

Table 6. Numerical results of thickness design for polyester in Example 2

Algorithms	H-J	Cai's method	0.618
Initial values (m)	Thickness (cm)	Thickness (cm)	Thickness (cm)
0.0005	0.7175		
0.001	0.7175	0.7167	0.7170
0.007	0.7175		
0.01	0.7175		

Table 7. Numerical results of thickness design for polypropylene in Example 2

Algorithms	H-J	Cai's method	0.618
Initial values (m)	Thickness (cm)	Thickness (cm)	Thickness (cm)
0.0005	0.70875		
0.001	0.70875	0.7091	0.7090
0.007	0.70875		
0.01	0.71125		

6. Concluding Remarks

According to the numerical results, we give some remarks:

(1) As for different initial values, Hooke-Jeeves' pattern search algorithm can find approximation of the optimal solution. But different initial values lead to different iterations and computation time, so convergence is much sensitive to the initial value. The Hooke-Jeeves' pattern search algorithm can not only solve the inverse problem of thickness design for single layer textile material, but also provide theoretical support and scientific explanation for textile material design.

(2) The 0.618 method is suitable for single peak function, so we should choose the initial interval which should include the minimum point; The Cai's direct search algorithm is suitable for single peak function, and we should choose two initial points which satisfy certain condition.

(3) The numerical results under two different low temperature conditions are both acceptable, as the thickness of textile under low temperature is between $0.5mm$ and $10mm$.

(4) We should further study the conditional well-posedness of the inverse problem and the theoretical convergence of the proposed algorithm.

(5)In this paper, we study the inverse problem of thickness design for single layer textile material under low temperature. We will continue to study the corresponding inverse problem for multi-layer textile material and give the results. As for dynamic model of coupled heat and moisture transfer, we can also study thickness design problems based on the clothing comfort, and corresponding results will be given in forthcoming papers.

Acknowledgments

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